Bayesian Hierarchical Occupancy Model

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1 Base Model 1 (no covariates)

We fit a model to our data, where we detected (1) or not detected (0) our species of interest ($y_{i,j}$) for site i and replicate j as,

$$\begin{aligned} y_{i,j} &\sim \text{Bernoulli}(z_i \times p_{i,j}) \\ z_i &\sim \text{Bernoulli}(\psi) \end{aligned}$$

1.0.1 Priors

$$\begin{aligned} p &\sim \text{Uniform}(1, 1) \\ \psi &\sim \text{Uniform}(1, 1) \end{aligned}$$

1.1 Alternate representation of the same model as explictly hierarhical

$$y_{ij} \sim \begin{cases} 0 & , z_1 = 0 \\ \text{Bernoulli}(p_{ij}) & , z_i = 1, \end{cases}$$

$$z_i \sim \text{Bernoulli}(\psi_i) \tag{1}$$

$$\tag{2}$$

1.1.1 Priors

$$p \sim \text{Uniform}(1,1)$$
$$\psi \sim \text{Uniform}(1,1)$$

1.2 JAGS syntax

```
model {
# Priors
psi~dunif(0,1)
p~dunif(0,1)

 # Loop over sites
for(i in 1:n.sites){
     z[i] ~ dbern(psi)
# Loop over occasions within sites
 for(j in 1:n.visits){
y[i,j] ~ dbern(p*z[i])
 } # j loop
} #i loop
```

```
} #End model
```

2 Model with covariates

We fit a model to our data, where we detected (1) or not detected (0) our species of interest (y_{ij}) for site i and replicate j. We link the probability of occupancy (ψ) and detection probability p with covariates of interest on the logit scale via the design matrices $\mathbf{X}$ and $\mathbf{W}$ along with their respective vector of coeficients, $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$.

$$\begin{aligned} y_{i,j} &\sim \text{Bernoulli}(z_i \times p_{i,j}) \\ z_i &\sim \text{Bernoulli}(\psi_i) \\ \text{logit}(\psi_i) &= \mathbf{X}_i\boldsymbol{\beta} \\ \text{logit}(p_{ij}) &= \mathbf{W}_{ij}\boldsymbol{\alpha} \end{aligned}$$

2.0.1 Priors

$$\begin{aligned} \beta_{p_1} &\sim \text{Logistic}(0, 1) \\ \alpha_{p_2} &\sim \text{Logistic}(0, 1) \end{aligned}$$

where, p_1 are the number of parameters to be estimated modeling ψ and p_2 are the number of parameters to be estimated modeling p.

2.1 JAGS syntax

```
model {
# Priors
  for(i in 1:n.beta){
    beta[i]~dlogis(0,1)
  }
  for(i in 1:n.alpha){
    alpha[i]~dlogis(0,1)
  }

# Loop over sites
  for(i in 1:n.sites){
   logit(psi[i]) <-  inprod(X[i,], beta)
   z[i] ~ dbern(psi[i])
  # Loop over occasions within sites
   for(j in 1:n.visits){
       logit(p[i,j]) <- inprod(W[i,], alpha)
       peff[i,j] <- p[i,j]*z[i]
       y[i,j] ~ dbern(peff[i,j])
   }# j loop
 } #i loop
} #End model
```